When $p=r/s$, $2m/(p^2-2)=2ms^2/(r^2-2s^2)$.....(3).

When $p=2(r+s)/(r+2s)$, $2m/(p^2-2)=m(r+2s)^2(r^2-2s^2)$.....(4).

Since (3) is integral, (4) is also.

Also solved by the *PROPOSER*.

GEOMETRY.

200. Proposed by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud, England.

Find the locus of eight points of contact of the four common tangents of two concentric coaxial ellipses.

Solution by G. B. M. ZERR, A. M., Ph.D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let $x^2/a^2+y^2/b^2=1$, and $x^2/c^2+y^2/d^2=1$, be the equations to the ellipses, $c>a$, $b>d$.

Since $xx_1/a^2+yy_1/b^2=1$, and $xx_2/c^2+yy_2/d^2=1$, are two equations for the same line, we get $x_2=c^2x_1/a^2$, $y_2=d^2y_1/b^2$.

$\therefore x_1^2/a^2+y_1^2/b^2=1$, and $c^2x_1^2/a^4+d^2y_1^2/b^4=1$, give

$$x_1=\frac{a^2\sqrt{[b^2-d^2]}}{\sqrt{[b^2c^2-a^2d^2]}}=a^2m \text{ (suppose)},\ y_1=\frac{b^2\sqrt{[c^2-a^2]}}{\sqrt{[b^2c^2-a^2d^2]}}=b^2n,$$

$$x_2=c^2m,\ y_2=d^2n.$$

Hence $mx\pm ny\pm1=0$ represents the four common tangents. While $(c^2m,\ d^2n)$; $(a^2m,\ b^2n)$; $(-a^2m,\ b^2n)$; $(-c^2m,\ d^2n)$; $(-c^2m,\ -d^2n)$; $(a^2m,\ -b^2n)$; $(c^2m,\ -d^2n)$ are the eight points of contact. These eight points are situated on an ellipse, which is the locus required.

Let $x^2/A^2+y^2/B^2=1$ be this ellipse. Then $c^2m/A^2+d^4n^2/B^2=1$, also $a^4m^2/A^2+b^4n^2/B^2=1$.

$$\therefore A^2=\frac{(b^4c^4-a^4d^4)m^2}{b^4-d^4}=\frac{b^2c^2+a^2d^2}{b^2+d^2},\ B^2=\frac{(b^4c^4-a^4d^4)n^2}{c^4-a^4}=\frac{b^2c^2+a^2d^2}{a^2+c^2}.$$

$$\therefore (b^2+d^2)x^2+(a^2+c^2)y^2=b^2c^2+a^2d^2 \text{ is the locus.}$$

201. Proposed by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud. England.

Two plane sections of a right circular cone have their major axes AA', aa' coplanar, and Aa on one generator equal to $A'a'$ on the other. The projections of the sections on any plane perpendicular to the axis are confocal.